inter-noise2003
The 32nd International Congress and Exposition on Noise Control Engineering
Jeju International Convention Center, Seogwipo, Korea,
August 25-28, 2003

[N905] Source arrays for the realization of broadband sound fields in ducts

Wontae Jeong, Soogab Lee
AeroAcoustics and Noise Control Laboratory, Seoul National University
301 Bldg., Shinlim-dong, Kwanak-gu, Seoul, Korea 151-744
Email address: wontae@snu.ac.kr

Philip Joseph
Institute of Sound and Vibration Research
University of Southampton, SO14 6HB, UK

ABSTRACT

In the analysis of broadband sound fields in ducts, for example in aero-engine ducts, the assumption of 'Equal Energy per Mode' (EEpM) and incoherence between modes is frequently made. The practical realization of such a sound fields is valuable as a means of, for example, allowing liner attenuation measurements obtained from measurements on different test rigs to be compared directly, or for allowing measurements results to be compared with computer predictions in which the EEpM assumption is made.
This paper describes a technique in which arrays of sound sources at the wall of a duct are driven by white noise signals to generate a sound field of prescribed modal energy distribution and modal coherence. The number of sources required for effective mode synthesis and the robustness of the processing as a function of frequency are also discussed.
An example is presented in which an EEpM, incoherent broadband sound field is generated up to a maximum non-dimensional frequency of ka=20 using 150 sources.

KEYWORDS: **Broadband Noise, Sound Synthesis, Equal Energy per Mode**

INTRODUCTION

There are many duct acoustic applications in which it is desirable to be able to generate a sound field of known modal content, for example, in the testing of liner performance, or as a calibrated sound field for the testing of different inlet geometries. The availability of a standardized in-duct sound field is available as a means of, for example, allowing the multi-mode performance of different liners obtained from different tests to be compared directly, or allowing measurement result to be meaningfully compared directly, or allowing measurements results to be meaningfully compared with computer predictions. Presently, there are no guidelines for the definition of such a sound field, although in analysis of broadband sound fields in ducts, the assumption of 'Equal Energy per Mode' (EEpM) is frequently made.

For the reason stated above, in-duct testing with an 'Equal Energy per Mode' sound field is desirable. However, creating such a sound field currently requires costly facilities. This paper is concerned with an alternative method that uses sound sources on the duct wall. Specification of the optimal filters driven by white noise signals has the advantage that it may be exactly reproducible.

Theory

Duct mode theory

At a single frequency, an incident mode without reflection can be written in the form

$$p_{mn}^{+} = e^{i\omega t} a_{mn} \Psi_{mn} e^{-i\alpha_{mn} kz}, \qquad \text{where,} \quad \alpha_{mn} = \sqrt{1-\left(\kappa_{mn}/k\right)^2} \qquad (1)$$

Here a_{mn} are the modal amplitudes, k is the free space wave number ω/c, c is the sound speed in the duct, ω is angular frequency. The wave numbers κ_{mn} are the modal eigenvalues that satisfy the hard walled boundary condition $J'(\kappa_{mn} a) = 0$, where J_m denotes the Bessel function of the first kind of order m, the prime signifies differentiation with respect to the argument, and a is the duct radius. And Ψ_{mn} are the normalized mode shape function [1] given by

$$\Psi_{mn} = J_{mn}(\kappa_{mn} r) e^{im\phi} / N_{mn} \tag{2}$$

Uncorrelated Equal Incident Sound Power Per Mode

The time-averaged sound power $\overline{W^{+}}(ka)$ incident at the open termination of the duct is given by

$$\overline{W^{+}} = \frac{i}{2\omega\rho} \int_{S} d\mathbf{x} p^{+} * (\mathbf{x}) \frac{\partial p^{+}(\mathbf{x})}{\partial z}, \qquad \mathbf{x} \in S \tag{3}$$

where ρ is the ambient air density in the duct. Upon substituting Eq. (3) the integration can be performed using the orthogonal property of the mode shape function such that

$$\overline{W_{mn}} = \frac{S}{2\rho c} \sum_{m,n} E\left\{ |a_{mn}|^2 \right\} \alpha_{mn} \qquad (\alpha_{mn} \text{ is real}) \tag{4}$$

If the sound power in each of the incident modes at a single frequency are assumed to be equal, from Eq. (4), and mode amplitudes uncorrelated (we treat the mode amplitude as random variables), this may by summarized as,

$$\left. \begin{array}{ll} E\left\{ |a_{mn}|^2 \right\} = \dfrac{2\rho c \varpi}{S} \alpha_{mn}^{-1} & (m,n) = (m',n') \\ E\left\{ a_{mn} a_{m'n'}^{*} \right\} = 0 & (m,n) \neq (m',n') \end{array} \right\} \tag{5}$$

In the broadband problem, we wish to compute the optimal source strengths $q_{ni}(\omega)$, obtained by Fourier Transform of the i'th record length of $q_{ni}(t)$ of duration T with cross spectral

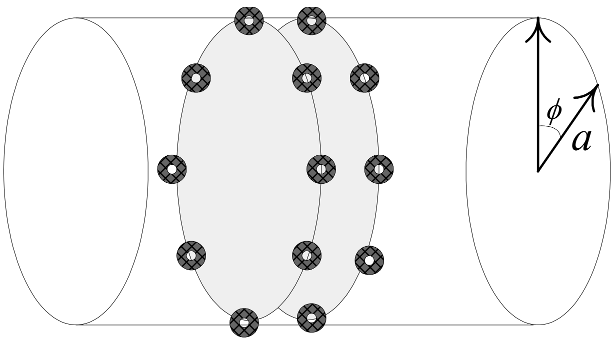

Fig. 1. Wall mounted sources are located in source rings with equiangular distance. Each Source rings placed with equidistance between them. Sources are driven to construct predefined sound field.

matrix S_{qq} which excites, in a least squares sense, a mode amplitude whose cross spectral matrix, from Eq. (5), is given by

$$\mathbf{S}_{aa} = \frac{2\rho c \varpi}{S}\begin{pmatrix} \alpha_1^{-1} & 0 & 0 & \cdot & \cdot & 0 \\ 0 & \alpha_2^{-1} & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \alpha_k^{-1} & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & \cdot & \cdot & \cdot & \cdot & \alpha_K^{-1} \end{pmatrix} \tag{6}$$

where $k = (m,n)$. Here, ϖ denotes the desired sound power per single mode in a unit frequency band.

Optimal Source Strength Spectral Matrix

We write the vector of mode amplitudes $\hat{\mathbf{a}}_i^T = \begin{bmatrix} a_{1i} & a_{2i} & a_{3i} & \cdot & \cdot & a_{Ni} \end{bmatrix}$ excited by the vector of source strengths $\hat{q}_i^T = \begin{bmatrix} q_{1i} & q_{2i} & q_{3i} & \cdot & \cdot & q_{Li} \end{bmatrix}$ in terms of a matrix of modal coupling factors G,

$$\hat{\mathbf{a}}_i = \mathbf{G}\mathbf{q}_i + \mathbf{e} \tag{7}$$

the optimal estimate of $\mathbf{q}_i = \mathbf{q}_{io}$ that minimizes the sum of squared errors $\mathbf{e}^{\mathbf{H}}\mathbf{e} = (\mathbf{a} - \hat{\mathbf{a}})^H(\mathbf{a} - \hat{\mathbf{a}})$ is given by

$$\mathbf{q}_{io} = \mathbf{G}^{+}\hat{\mathbf{a}}_i \tag{8}$$

where $\mathbf{G}^{+} = \left[\mathbf{G}^H\mathbf{G}\right]^{-1}\mathbf{G}^H$ and denotes the pseudo-inverses of $\mathbf{G}$. The source-strength of cross-spectral matrix may be written

$$\mathbf{S}_{qqo} = \lim_{T\to\infty} E\left\{\frac{1}{T}\mathbf{q}_{io}\mathbf{q}_{io}^H\right\} \tag{9}$$

Substituting Eq. (8) gives the optimal cross-spectral matrix,

$$\mathbf{S}_{qqo} = \mathbf{G}^{+}\mathbf{S}_{aa}\mathbf{G}^{+H} \tag{10}$$

The least-squares best estimate for the mode-amplitude cross-spectral matrix is given by

$$\hat{\mathbf{S}}_{aa} = \mathbf{G}\mathbf{S}_{qqo}\mathbf{G}^{H} \tag{11}$$

Realization, Optimal Shaping Filters

We assume that the volume velocities can be generated by a square matrix of shaping filters $\mathbf{H}$ driven by a number of white noise input signals $\mathbf{x}$

$$\mathbf{q}_{so} = \mathbf{H}\mathbf{x} \tag{12}$$

$$q_{i,so}(\omega) = H_{i,1}(\omega)x_1(\omega) + H_{i,2}(\omega)x_2(\omega) + \cdots + H_{i,N}(\omega)x_N(\omega)$$

The source cross-spectral matrix is then given by

$$\mathbf{S}_{qqo} = \mathbf{H}\mathbf{S}_{xx}\mathbf{H}^{H} \tag{13}$$

For simplicity we assume $\mathbf{x}$ comprise uncorrelated white noise signals with variance σ^2, then $\mathbf{S}_{qqo}$ becomes

$$\frac{\mathbf{S}_{qqo}}{\sigma^2} = \mathbf{H}\mathbf{H}^{H} \tag{14}$$

The shaping filter matrix $\mathbf{H}$ can be acquired by decomposing Hermitian matrix $\mathbf{S}_{qqo}$.

SIMULATION RESULTS

We now present computer predictions at a single (non-dimensional) frequency of $ka = 15$. At this frequency there are total of K modes, the highest spinning mode that can be propagate is $m_{\max} = 13$, and the largest number of radial modes with common 'm' value is $n_{\max} = 5$

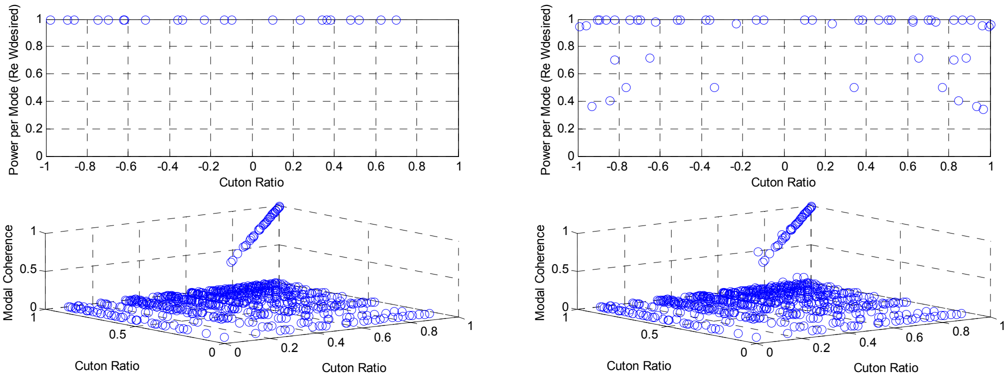

Fig. 2. Non-dimensional frequency ka=15 for both graph. For left figure 2a, 5 source rings with 27 sources per rings are used to perfect EEpM sound field construction. For right figure, 4 source rings with 16 sources are failed to construct EEpM sound field.

for $m=0$. The sampling (Nyquist) theorem suggests that perfect sound field construction is possible (i.e. without aliasing and hence modal spillover) for numbers of sources in a given ring, $M_{source} = 2m_{\max} + 1$, and for numbers of rings $N_{ring} = n_{\max}$. These criteria applied to the current problem suggests that the use of 5 source rings separated half a wavelength apart, each ring containing 27 sources. Predicted results for this case are shown figure 2a below. Upper part of figure 2a shows $\overline{W_{nm}} / \varpi$ plotted against the modal cuton ratio α_{mn}. ($\alpha = 0$ denotes a mode just cuton and $\alpha \to 1$ denotes a mode very well cuton). Negative α values denote modes spinning in the opposite direction. Lower part of figure 2b shows the modal coherence

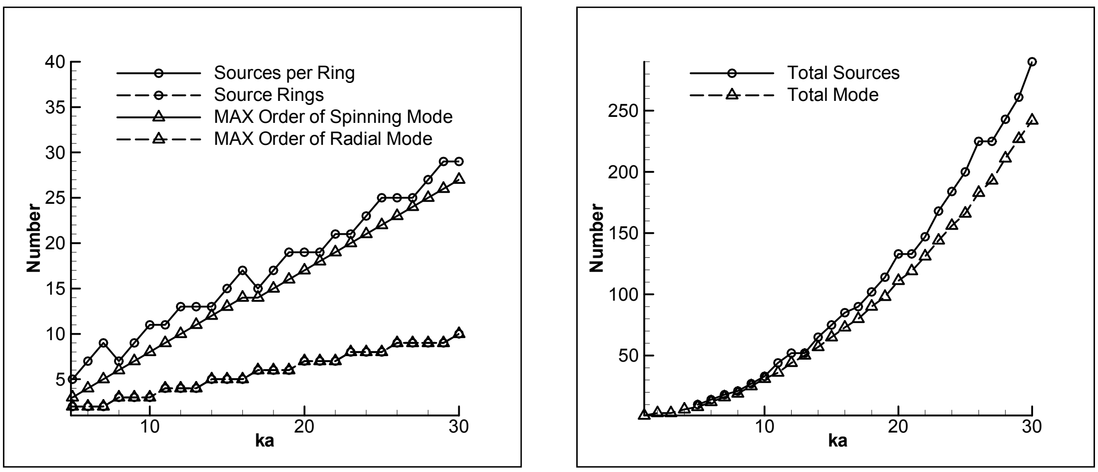

Fig. 3. Left figure 3a shows the optimum number of sources per ring and source rings to construct EEpM Sound field vs ka. Right figure 3a shows total source number vs ka. Both graph compares number of sources with number of modes.

$\gamma^2_{mnm'n'}$ defined by

$$\gamma_{mnm'n'} = \frac{E\left\{\left|a_{mn}a^*_{m'n'}\right|^2\right\}}{E\left\{\left|a_{mn}\right|^2\right\}E\left\{\left|a_{m'n'}\right|^2\right\}} \tag{15}$$

plotted against cuton ratio (only positive α values are shown for compactness).
The predictions show exactly equal energy per mode has been excited, with different modes being perfectly incoherent (except of course along the diagonal in figure 2a since each mode must be perfectly coherent with itself).
Figure 3b shows number of optimal sources for perfect sound field construction is slightly above the number of total modes. From figure 3a, we can see that number of total spinning mode $2m_{\max}+1$ is much more than the number of sources in one source ring. With insufficient number of sources per ring, perfect sound field construction is enabled. For, spinning mode is coupled with axial wave number. Perfect construction is enabled with enough number of source rings in axial direction.

Adding small noise in S_{qq} matrix makes estimated mode amplitude matrix S_{aa} unsatisfactory. Figure 4 shows the effect of adding –40dB random noise to S_{qq} matrix. From Eq. (11), we can notice the condition number of G matrix is related with magnification of errors in S_{aa} matrix [2]. As non-dimensional frequency ka goes high, number of mode increases. It makes condition number of G high. However, in real situation, small damping in wall makes near evanescent mode decay. If we do not concern about near-evanescent mode,

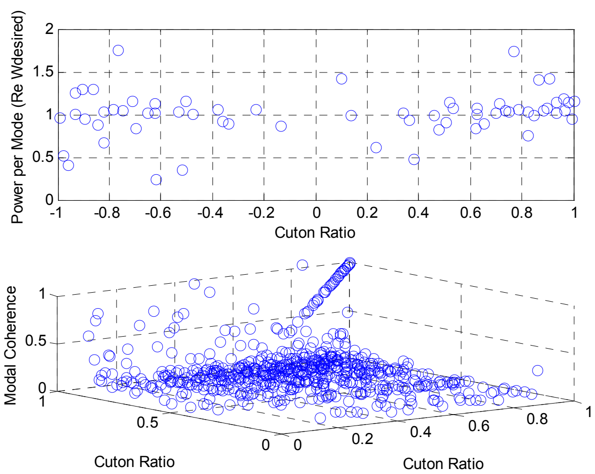

Fig. 4. ka=15, 27 sources per source ring, 5 source rings. Adding -40dB noise in the Sqq matrix cause construction of EEpM sound field totally unsatisfactory.

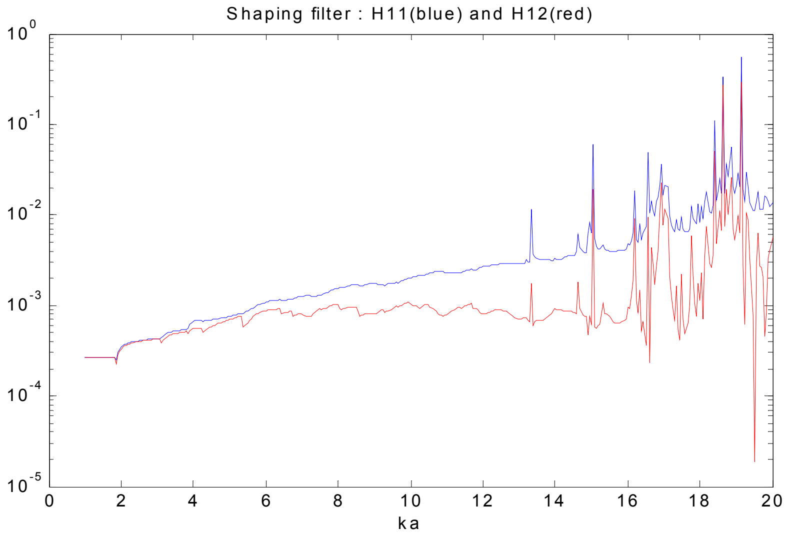

Fig. 5. Example of Shaping filters, 19 sources per rings, 8 source rings with 1m space.

the condition number will be better.

We get the shaping filter for the case that ka ranges from 0.5 to 20, radius of duct is 1 meter, 19 sources per rings, 8 rings in duct, and distance between each source ring is 1m, without adding noise. It seems like that as ka increases, the magnitude of shaping filter increases. It means volume velocity source in duct wall be large in high frequency.

CONCLUSIONS

The technique for generating broadband sound field of equal energy per mode and modal incoherence characteristics with wall-mounted sources is described. Optimal number of sources compared to Nyquist theorem and robustness to noise is also discussed. Pressure signal in the duct from wall-mounted sources driven by white noise can be acquired using shaping filters. The characteristic of EEpM pressure signal will be studied later.

Acknowledgement: This work is supported by International Cooperative Research Program of Korea Institute of Science and Technology Evaluation and Planning (KISTEP) and the Brain Korea 21 Project.

REFERENCES

1. P. M. Mores, *Theoretical Acoustics* (Princeton, New Jersey, 1987)
2. David R. Hill, *Experiments in computational matrix algebra* (McGraw Hill, New York, 1988)